Cambridge International A Level

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MATHEMATICS **9709/32**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $3^2(2x-1)^2 > (x+4)^2$, or corresponding quadratic equation, or pair of linear equations/inequalities $3(2x-1) = \pm(x+4)$	**B1**	$35x^2 - 44x - 7 = 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Allow for reasonable attempt at factorising e.g. $(5x - 7)(7x + 1)$
	Obtain critical values $x = \frac{7}{5}$ and $x = -\frac{1}{7}$	**A1**	Accept 1.4 and -0.143 or better for penultimate A mark
	State final answer $x > \frac{7}{5}$, $x < -\frac{1}{7}$	**A1**	‘and’ is A0, $\frac{7}{5} < x < -\frac{1}{7}$ is A0. Must be exact values. Must be strict inequalities in final answer
	Alternative		
	Obtain critical value $x = \frac{7}{5}$ from a graphical method	**B1**	or by inspection, or by solving a linear equation or an inequality
	Obtain critical value $x = -\frac{1}{7}$ similarly	**B2**	
	State final answer $x > \frac{7}{5}$ or $x < -\frac{1}{7}$ or equivalent	**B1**	[Do not condone $\geqslant$ for $>$, or $\leqslant$ for $<$.]
		4	

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Question	Answer	Marks	Guidance
2	Use trig formula and obtain an equation in $\sin\theta$ and $\cos\theta$	**M1***	Condone sign error in expansion and/or omission of "$+\cos\theta$" $\sin\theta\cos 30° - \cos\theta\sin 30° + \cos\theta = 2\sin\theta$
	Obtain an equation in $\tan\theta$	**M1(dep*)**	e.g. $\tan\theta = \frac{1-\sin 30°}{2-\cos 30°}$ Can be implied by correct answer following correct expansion. Otherwise need to see working
	Obtain $\tan\theta = 1/\left(4-\sqrt{3}\right)$, or equivalent	**A1**	$\frac{4+\sqrt{3}}{13}$, $0.4409....$ (2 s.f or better)
	Obtain final answer $\theta = 23.8°$ and no others in range	**A1**	At least 3 sf $(23.7939....)$ ignore extra values outside range
		4	

Question	Answer	Marks	Guidance
3(i)	Integrate by parts and reach $a\dfrac{\ln x}{x^2} + b\int\dfrac{1}{x}.\dfrac{1}{x^2}\,\mathrm{d}x$	**M1***	
	Obtain $\pm\dfrac{1}{2}\dfrac{\ln x}{x^2} \pm \int\dfrac{1}{x}.\dfrac{1}{2x^2}\,\mathrm{d}x$, or equivalent	**A1**	
	Complete integration correctly and obtain $-\dfrac{\ln x}{2x^2} - \dfrac{1}{4x^2}$, or equivalent	**A1**	Condone without '$+C$' ISW
		3	

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Question	Answer	Marks	Guidance
3(ii)	Substitute limits correctly in an expression of the form $a\frac{\ln x}{x^2}+\frac{b}{x^2}$ or equivalent	M1(dep*)	$-\frac{1}{8}\ln 2-\frac{1}{16}+\frac{1}{4}$
	Obtain the **given answer** following full and **exact** working	A1	The step $\ln 2=\frac{1}{2}\ln 4$ or 2ln2 = ln4 needs to be clear.
		2	

Question	Answer	Marks	Guidance
4	Substitute and obtain 3-term quadratic $3u^2+4u-1=0$, or equivalent	B1	e.g. $3\left(e^x\right)^2+4e^x-1=0$
	Solve a 3 term quadratic for u	M1	Must be an equation with real roots
	Obtain root $\left(\sqrt{7}-2\right)/3$, or decimal in [0.21, 0.22]	A1	Or equivalent. Ignore second root (even if incorrect)
	Use correct method for finding x **from a positive value of** e^x	M1	Must see some indication of method: use of $x=\ln u$
	Obtain answer $x = -1.536$ only	A1	CAO. Must be 3 dp
		5	

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Question	Answer	Marks	Guidance
5(i)	Use product rule on a correct expression	M1	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	A1	$\frac{dy}{dx}=\ln(8-x)-\frac{x}{8-x}$
	Equate derivative to 1 and obtain $x=8-\frac{8}{\ln(8-x)}$	A1	**Given answer:** check carefully that it follows from correct working
			Condone the use of *a* for *x* throughout
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at x = 2.9 and x = 3.1	M1	$8-\frac{8}{\ln 5.1}=3.09>2.9,\quad 8-\frac{8}{\ln 4.9}=2.97<3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	A1	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

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Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $x_{n+1} = 8 - \dfrac{8}{\ln(8 - x_n)}$ correctly to find at least two successive values. SR: Clear successive use of 0, 1, 2, 3 etc., or equivalent, scores M0.	M1	3, 3.0293, 3.0111, 3.0225, 3.0154, (3.0198) 2.9, 3.0897, 2.9728, 3.0460, 3.0006, 3.290, 3.0113, 3.0223, 3.0155 3.1, 2.9661, 3.0501, 2.9980, 3.0305, 3.0103, 3.0229, 3.0151 Allow M1 if values given to fewer than 4 dp
	Obtain final answer 3.02	A1	
	Show sufficient iterations to at least 4 d.p. to justify 3.02 to 2 d.p., or show there is a sign change in the interval (3.015, 3.025)	A1	Must have two consecutive values rounding correctly to 3.02
		3	

Question	Answer	Marks	Guidance
6	State equation $\frac{dy}{dx} = k\frac{y^2}{x}$, or equivalent	**B1**	SC: If $k = 1$ seen or implied give B0 and then allow B1B1B0M1, max 3/8.
	Separate variables correctly and integrate at least one side	**B1**	$\int \frac{k}{x}dx = \int \frac{1}{y^2}dy$ Allow with incorrect value substituted for k
	Obtain terms $-\frac{1}{y}$ and $k \ln x$	**B1 + B1**	Incorrect k used scores max. B1B0
	Use given coordinates correctly to find k and/or a constant of integration C in an equation containing terms $\frac{a}{y}$, $b\ln x$ and C	**M1**	SC: If an incorrect method is used to find k, M1 is allowable for a correct method to find C
	Obtain $k = \frac{1}{2}$ and $c = -1$, or equivalent	**A1 + A1**	$\frac{1}{2}\ln x = 1 - \frac{1}{y}$ A0 for fortuitous answers.
	Obtain answer $y = \frac{2}{2 - \ln x}$, or equivalent, and ISW	**A1**	$y = \frac{-1}{-1 + \ln\sqrt{x}}$
			SC: MR of the fraction. $\frac{dy}{dx} = k\frac{y^2}{x^2}$ B1 Separate variables and integrate B1 $\frac{-1}{y} = \frac{-k}{x}(+C)$ B1+B1 Substitute to find k and/or c M1 $k = \frac{e}{2(e-1)}$, $c = \frac{2-e}{2(e-1)}$ A1+A1 Answer A0
		8	

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Question	Answer	Marks	Guidance
7(i)	Use correct quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx} = \frac{-3\sin x(2+\sin x) - 3\cos x \cos x}{(2+\sin x)^2}$ Condone invisible brackets if recovery implied later.
	Equate numerator to zero	**M1**	
	Use $\cos^2 x + \sin^2 x = 1$ and solve for $\sin x$	**M1**	$-6\sin x - 3 = 0 \Rightarrow \sin x = \ldots$
	Obtain coordinates $x = -\pi/6$ and $y = \sqrt{3}$ ISW	**A1 + A1**	From correct working. No others in range
			SR: A candidate who only states the numerator of the derivative, but justifies this, can have full marks. Otherwise they score M0A0M1M1A0A0
		6	
7(ii)	State indefinite integral of the form k ln (2 + sin x)	**M1***	
	Substitute limits correctly, equate result to 1 and obtain 3 ln (2 + sin a) – 3 ln 2 = 1	**A1**	or equivalent
	Use correct method to solve for a	**M1(dep*)**	Allow for a correct method to solve an incorrect equation, so long as that equation has a solution. $1 + \frac{1}{2}\sin a = e^{1/3} \Rightarrow a = \sin^{-1}\left[2\left(e^{1/3} - 1\right)\right]$ Can be implied by 52.3°
	Obtain answer a = 0.913 or better	**A1**	Ignore additional solutions. Must be in radians.
		4	

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Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{1-2x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	**B1**	
	Use a correct method for finding a constant M1 is available following a single slip in working from their form but no A marks (even if a constant is "correct")	**M1**	$7=A+2B$ $-15=-4A-5B-2C$ $8=4A+2B+C$
	Obtain one of $A=1$, $B=3$, $C=-2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	[Mark the form $\dfrac{A}{1-2x}+\dfrac{Dx+E}{(2-x)^2}$, where $A=1$, $D=-3$ and $E=4$, B1M1A1A1A1 as above.]		
		5	

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Question	Answer	Marks	Guidance
8(ii)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2-x)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2-x)^{-2}$ or $\left(1-\frac{1}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3ft**	$1+2x+4x^2$ The ft is on A, B, C. $\frac{3}{2}+\frac{3}{4}x+\frac{3}{8}x^2$ $-\frac{1}{2}-\frac{1}{2}x-\frac{3}{8}x^2$
	Obtain **final** answer $2+\frac{9}{4}x+4x^2$	**A1**	
	[For the A, D, E form of fractions give M1A2ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.]		[The ft is on A, D, E.]
		5	

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Question	Answer	Marks	Guidance
9(a)(i)	Multiply numerator and denominator by 1 + 2i, or equivalent	**M1**	Requires at least one of $2+10\mathrm{i}+12\mathrm{i}^2$ and $1-4\mathrm{i}^2$ together with use of $\mathrm{i}^2=-1$. Can be implied by $\frac{-10+10\mathrm{i}}{5}$
	Obtain quotient $-2+2\mathrm{i}$	**A1**	
	Alternative		
	Equate to $x+\mathrm{i}y$, obtain two equations in x and y and solve for x or for y	**M1**	$x+2y=2$, $y-2x=6$
	Obtain quotient $-2+2\mathrm{i}$	**A1**	
		2	
9(a)(ii)	Use correct method to find either r or θ	**M1**	If only finding θ, need to be looking for θ in the correct quadrant
	Obtain $r=2\sqrt{2}$, or exact equivalent	**A1ft**	ft their $x+\mathrm{i}y$
	Obtain $\theta=\frac{3}{4}\pi$ from exact work	**A1ft**	ft on $k(-1+\mathrm{i})$ for $k>0$ Do not ISW
		3	

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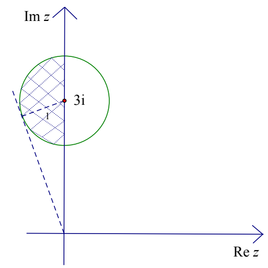
Question	Answer	Marks	Guidance
9(b)	Show a circle with centre 3i	**B1**	
	Show a circle with radius 1	**B1ft**	Follow through their centre provided not at the origin For clearly unequal scales, should be an ellipse
	All correct with even scales and **shade** the correct region	**B1**	
	Carry out a correct method for calculating greatest value of arg z	**M1**	e.g. $\arg z = \dfrac{\pi}{2} + \sin^{-1}\dfrac{1}{3}$
	Obtain answer 1.91	**A1**	
		5	

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Question	Answer	Marks	Guidance
10(i)	Substitute for **r** and expand the scalar product to obtain an equation in λ	M1*	e.g. $3(5+\lambda)+(-3-2\lambda)+(-1+\lambda)=5\ \ (2\lambda=5-11)$ or $3(4+\lambda)+1(-5-2\lambda)+(-1+\lambda)=0$ Must attempt to deal with **i** + 2**j**
	Solve a linear equation for λ	M1(dep*)	
	Obtain $\lambda = -3$ and position vector $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ for A	A1	Accept coordinates
		3	
10(ii)	State or imply a normal vector of p is 3**i** + **j** + **k**, or equivalent	B1	
	Use correct method to evaluate a scalar product of relevant vectors e.g. (**i** – 2**j** + **k**).(3**i** + **j** + **k**)	M1	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	M1	$\cos\theta = \dfrac{2}{\sqrt{6}\sqrt{11}}$ Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	A1	Or better

Question	Answer	Marks	Guidance
10(ii)	**Alternative 1**		
	Use of a point on l and Cartesian equation $3x+y+z=5$ to find distance of point from plane e.g. $B(5,-3,-1)$ $d=\frac{3\times5-3-1-5}{\sqrt{9+1+1}}$	**M1**	
	$=\frac{6}{\sqrt{11}}\quad(=1.809...)$	**A1**	
	Complete method to find angle e.g. $\sin\theta=\dfrac{d}{AB}$	**M1**	
	$\theta=\sin^{-1}\left(\dfrac{6}{\sqrt{11}\sqrt{54}}\right)=0.249$	**A1**	Or better
	Alternative 2		
	State or imply a normal vector of p is 3**i** + **j** + **k**, or equivalent	**B1**	
	Use correct method to evaluate a vector product of relevant vectors e.g. (**i** – 2**j** + **k**)**x**(3**i** + **j** + **k**)	**M1**	$3\mathbf{i}-2\mathbf{j}+7\mathbf{k}$
	Using the correct process for calculating the moduli, divide the vector product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\sin\theta=\dfrac{\sqrt{3^2+2^2+7^2}}{\sqrt{11}\sqrt{6}}$. Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	**A1**	Or better
		4	

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Question	Answer	Marks	Guidance
10(iii)	Taking the direction vector of the line to be $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, state a relevant equation in a, b, c, e.g. $3a + b + c = 0$	**B1**	
	State a second relevant equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c = 3 : -2 : -7$, or equivalent	**A1**	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_A$ Requires '$\mathbf{r} = \ldots$.'
	Alternative		
	Attempt to calculate the vector product of relevant vectors, e.g. $(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain correct product, e.g. $3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$	**A1**	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_{A.}$ Requires " $\mathbf{r} = \ldots$."
		4	